Modification of the Generalized tanh Method with sech Function for Generalized Hamiltonian Equations

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We modify the generalized tanh method with the sech function, so as to obtain a new class of the soliton-like solutions to a coupled set of generalized Hamiltonian equations.

The generalized tanh method and its extensions [1–3] are a tool to directly construct the tanh-profiled soliton-like solutions for certain equations of mathematical physics.

The sech function, like tanh, does not diverge at infinity either (for real arguments). We hereby try to modify the generalized tanh method by assuming that certain soliton-like solutions, physically localized, be of the form

$$u(x,t) = \sum_{l=0}^{L} \mathscr{A}_l(x,t) \cdot \tanh^l[\Psi(x,t)] + \sum_{j=0}^{J} \mathscr{B}_j(x,t) \cdot \operatorname{sech}[\Psi(x,t)] \cdot \tanh^j[\Psi(x,t)]\,, \quad (1)$$

where L and J are the integers determined via the balance of the highest-order contributions from both the linear and nonlinear terms of the original equation, while $\mathscr{A}_l(x,t)$'s, $\mathscr{B}_j(x,t)$'s, and $\Psi(x,t)$ are differentiable functions. $\Psi(x,t)$ should be real to make sure that the soliton profiles hold.

It is noted that in the generalized tanh method, the soliton-like solutions only come from the first summation in Ansatz (1). What we do is to make the power series more complete via the introduction of the second summation.

We now apply the above modification to a coupled set of generalized Hamiltonian equations,

$$u_t = u_x + 2v\,, \quad (2)$$

$$v_t = 2\varepsilon u v\,, \quad \text{where} \quad \varepsilon = \pm 1\,, \quad (3)$$

which has an infinite number of conserved densities, the nondegenerate Hamiltonian structure, certain Bäcklund transformations and exact solutions [4–8].

We use Ansatz (1) for both $u(x,t)$ and $v(x,t)$, and then equate to zero the coefficients of like powers of sech and tanh to get the explicit expressions for $\mathscr{A}_l(x,t)$'s and $\mathscr{B}_j(x,t)$'s. For simplicity, we consider a trial $\Psi(x,t) = \sum_{j=0}^{3} \Psi_j(x)\,t^j$, and after computerized symbolic computation end up with $\Psi(x,t) = \beta t + \gamma(x)$, where β is a real, non-zero constant, while $\gamma(x)$ is a real, differentiable function with $\gamma_x(x) \not\to \pm\infty$ when $x \to \pm\infty$. Thus, we obtain a new class of the soliton-like solutions as

$$u(x,t) = \frac{1}{2}\{\pm i|\beta|\operatorname{sech}[\beta t + \gamma(x)] - \varepsilon\beta\tanh[\beta t + \gamma(x)]\}\,, \quad (4)$$

$$v(x,t) = \frac{\gamma_x(x) - \beta}{4} \cdot \{\pm i|\beta|\operatorname{sech}[\beta t + \gamma(x)] \cdot \tanh[\beta t + \gamma(x)] + \varepsilon\beta\operatorname{sech}^2[\beta t + \gamma(x)]\}\,. \quad (5)$$

This class is different from that in [2, 7, 8]. It can also be reduced to solitary waves when $\gamma(x) = \alpha x + \delta$, where α and δ are real constants.

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[1] B. Tian and Y. T. Gao, Computer Phys. Comm. **95,** 139 (1996).
[2] B. Tian and Y. T. Gao, to appear in Appl. Math. Lett. (1997).
[3] B. Tian and Y. T. Gao, to appear in Chaos, Solitons & Fractals (1997).
[4] G. Tu, Phys. Lett. A **94,** 340 (1983).
[5] W. Ma, Kexue Tongbao **32,** 1003 (1987).
[6] Z. Chen, Comm. on Appl. Math. & Comput. **4,** 71 (1990).
[7] W. Ma, J. Fudan Univ. (Natural Sci.) **33,** 319 (1994).
[8] Y. Li and C. Tian, Kexue Tongbao (Lett.) **29,** 1556 (1984).

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